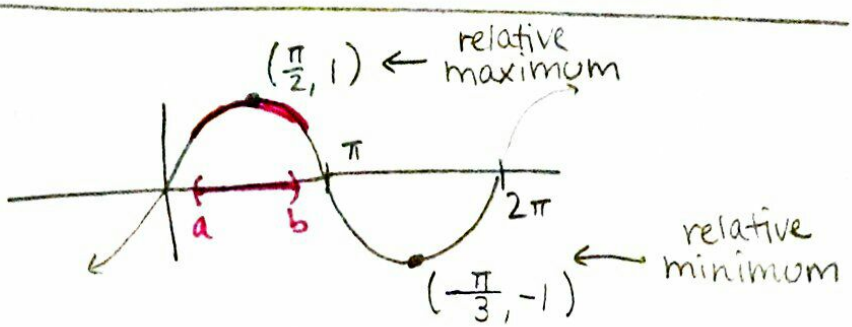
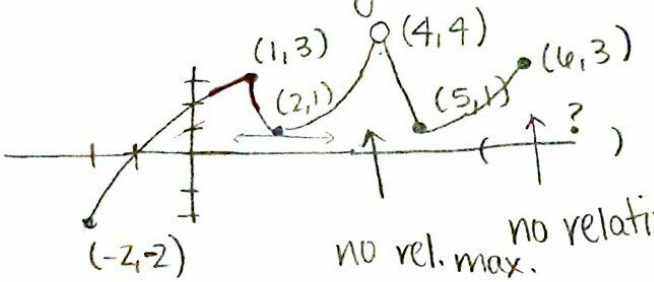
Lesson 17: Relative Extrema and Critical Values

Ex $f(x) = \sin x$



Def $f(x)$ has a relative maximum (relative minimum) at $(c, f(c))$ if there is an open interval (a,b) containing c in which $f(c)$ is the maximum (minimum) y-value.

Ex 1



relative max: (1,3)
relative min: (2,1), (5,1)

Notice: At each relative extrema, $f'(c) = 0$ or undefined. (In fact, this is the only place where relative extrema can occur!)

Def If $f'(c) = 0$ or undefined, and $f(c)$ exists, then c is a critical number (or critical value, CV)

Ex 2 $f(x) = \frac{x^2}{x+1}$. Find CV's.

$$f'(x) = \frac{(x+1)(2x) - x^2(1)}{(x+1)^2} = \frac{2x^2+2x-x^2}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$$

$f'(x) = 0$ if $x^2+2x = 0$

$x(x+2) = 0$

$\boxed{x = 0, \; x = -2}$

$f'(x)$ is undefined if $(x+1)^2 = 0$

$x = -1$

$f(-1)$ undefined

Ex 3 $y = x^3 - 3x$. Find CV's.

$y' = 3x^2 - 3 \overset{\text{set}}{=} 0$

$3(x^2 - 1) = 0$

$3(x+1)(x-1) = 0$

$\boxed{x = -1 \text{ or } x = 1}$

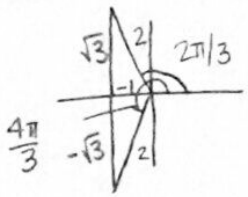
Ex 4 $y = 2\sin(3x) + 3x$ on $(0, \pi)$

$y' = 6\cos(3x) + 3 \overset{\text{set}}{=} 0$

$6\cos(3x) = -3$

$\cos(3x) = \frac{-3}{6} = -\frac{1}{2}$

$\cos\theta = -\frac{1}{2}$



$\theta = \frac{2\pi}{3} + 2\pi n$, n an integer

$\theta = \frac{4\pi}{3} + 2\pi n$

So $\cos(3x) = -\frac{1}{2}$ when $\frac{3x}{3} = \frac{\frac{2\pi}{3} + 2\pi n}{3}$ or $\frac{3x}{3} = \frac{\frac{4\pi}{3} + 2\pi n}{3}$

$x = \frac{2\pi}{9} + \frac{2\pi}{3}n$ $\quad$ $x = \frac{4\pi}{9} + \frac{2\pi}{3}n$

In the interval $(0, \pi)$: $x = \frac{2\pi}{9}, \frac{2\pi}{9} + \frac{2\pi}{3}, \frac{4\pi}{9}$

$\boxed{x = \frac{2\pi}{9}, \frac{8\pi}{9}, \frac{4\pi}{9}}$